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RIGOROUS METHOD FOR THE DETERMINATION OF THE MEAN CAMBER LINE FOR PROFILES OF UNKNOWN CAMBER

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16. Abstract

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RIGOROUS METHOD FOR THE DETERMINATION OF THE MEAN CAMBER LINE FOR PROFILES OF UNKNOWN CAMBER

K. Bausch*

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I. INTRODUCTION

The problem of deforming a symmetric profile is usually solved by distributing the thickness function y - f(x) of the drop with the dimensionless profile chord l over the desired

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* Berlin-Adlershof
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camber line with the dimensionless chord length 1. This is done by making a correspondence between points of the camber line g(x) and the thickness distribution f(x) having the same abscissa value, and the drop thickness f(x) is plotted to both sides from the selected chamber point along the camber normal. Analytically, we have the following relationships for the points X_0, Y_0 and X_u, Y_u of the contour of the deformed profile:

$$\left.\begin{aligned} X &= x \mp f(x) \cdot \frac{g'(x)}{\sqrt{1+g'^2(x)}} \\ Y &= g(x) \pm f(x) \cdot \frac{1}{\sqrt{1+g'^2(x)}} \end{aligned}\right\} . \qquad (1)$$

where the upper sign applies for the upper (subscript 0) and the lower sign applies for the lower contour point (subscript u). The deformation of a given drop using a specified camber line can be done easily using graphical and computational methods point by point.

However, it is more difficult to solve the reverse problem, that is for a given profile with an unknown curvature, to determine the elements which created it, the symmetric drop and the camber line. Only a single point of the desired camber line is known, the profile trailing edge. Nothing is known about the correspondence between points on the profile topside and the profile bottomside in the sense of Equation (1). In addition, the still unknown camber line must satisfy the requirement, because of the construction law of the form profiles, that the camber normal is bisected by the camber line.

II. DISCUSSION OF APPROXIMATION METHODS IN USE

The most used methods for determining the camber line have been developed without consideration of this basic characteristic of the camber line, as far as the author knows, and therefore are all better or worse approximations.

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The roughest approximations result when the connection curve of the centers of the profile secants, which are approximately perpendicular to the profile chord direction, are assumed to be the camber line. Figure 1 shows immediately that the segments P_1O and OP_2 of the normal are not of equal length along the false camber line, because we have

$$P_1O < \frac{s}{2} \quad \text{and} \quad OP_2 > \frac{s}{2},$$

where s is the length of the corresponding secant which is bisected at O.

The true camber lines are much better approximated by the curves which connect the circular or chord centers of the envelope circles which are drawn inside the profile. However, the deviations at the points of large curvature of the camber line are still large. Figure 2 shows the reason for this. If P_1Q_1, P_2Q_2, P_3Q_3 are three camber normals in sequence at the camber line point O_1, O_2, O_3, then the envelope circles which pass through Q_1, Q_2, Q_3 do not simultaneously pass through P_1, P_2, P_3. Instead, they pass through the other profile points, P_1', P_2', P_3'. Both their centers M_1, M_2, M_3 as well as the centers of the osculating chords $P_1'Q_1$, $P_2'Q_2$, $P_3'Q_3$ lie outside of the camber line. We will briefly discuss the mathematical proof: the profile tangents at the end points P_1 and Q_1 of the camber normal through O_1 always have unequal angles with it, because due to the camber line curvature, in the triangles $P_1P_2R_1$ and $Q_1Q_2R_2$ where $R_1P_2 = R_2Q_2 = \Delta y$ are the increases of the camber normal P_2Q_2 which passes through O_2 in comparison with the camber normal P_1Q_1. The sides P_1R_1 and Q_1R_2 have different lengths, that is $P_1R_1 = [\rho(x) + f(x)]\,\Delta\varphi$; $Q_1R_2 = [\rho(x) - f(x)]\,\Delta\varphi$ ($\rho(x)$ = radius of curvature of camber line at point x. Therefore, we also have

$$\measuredangle P_2P_1R_1 < \measuredangle Q_2Q_1R_2,$$

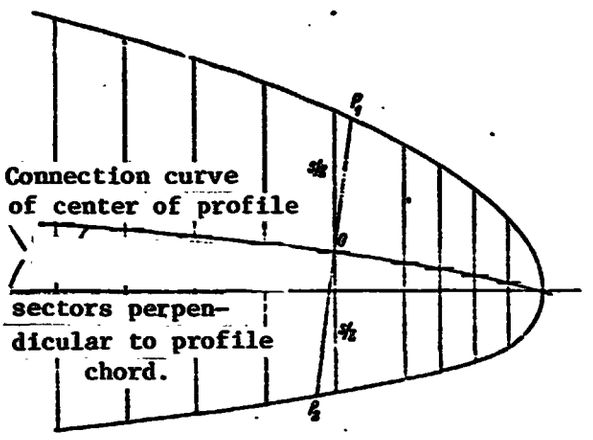

Figure 1. The connection line of the centers of parallel profile chords is the roughest camber line approximation (however it is insufficient).

and the triangle $S_N P_1 Q_1$ which occurs for the limit transition $O_2 \rightarrow O_1$ and which is formed by the profile tangents in P_1 and Q_1 with the camber normal at O_1 will never be an isosceles triangle. In contrast to this, the contact triangle $S_K P_1' Q_1$ which is formed by the tangents to the envelope circle M_1 is isosceles; $P_1' Q_1$ therefore cannot be a camber normal and its center Z_1 cannot be a camber point, because each rotation of $P_1' Q_1$ around Z_1 will disturb the fact that $P_1' Z_1$ and $Z_1 Q_1'$ are equal. / 12

However, the envelope circle center M_1 cannot belong to the camber line. The line parallel to $P_1' Q_1$ through M_1 will be a normal to the curve of all envelope centers because of symmetry, but their segments $M_1 P_1''$ and $M_1 Q_1''$ up to the profile are not of equal length because of the differences in the change rate of the profile tangent directions all along the upperside and the bottomside. The curve of the envelope centers therefore also cannot be identical with the camber line.

III. EXACT TRIAL SOLUTION

The attempts discussed above have shown that it is not possible to solve the problem of camber line determination using elementary geometry. On the other hand, a simple analysis shows that differential geometry methods must lead to an exact solution. If we consider the definition of the camber line given at the beginning as the curve whose normal is divided into equal parts by the profile, then this means conversely that the camber line is composed of the line elements which are formed by the center

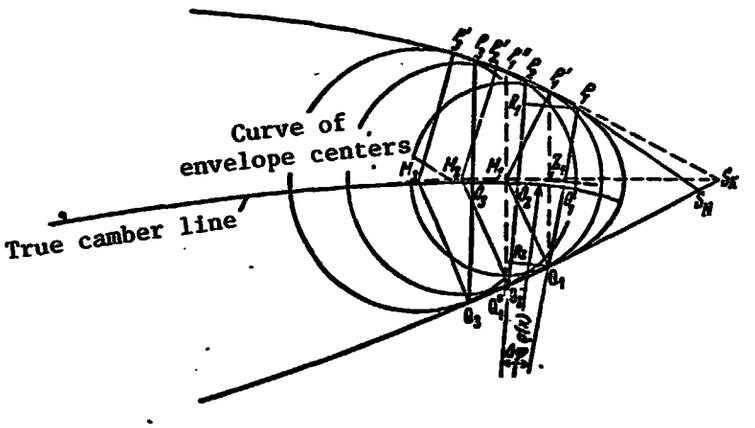

Figure 2. Comparison of true camber line and envelope circle method.

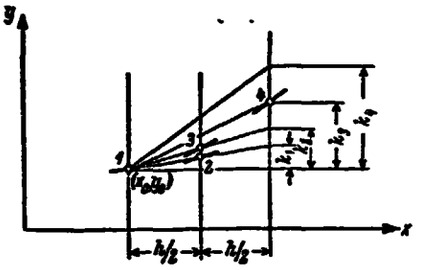

Figure 3. Diagram of the Runge-Kutta integration method.

normals of certain profile secants. If we consider the collection of all profile secants and all line elements normal to them and which bisect them, then we must collect out of this ∞^2 -multiple family of line elements, ∞^1 to make up a particular solution, that is, the desired camber line. Analytically this is done using the following trial solution.

Let $y_0 = h_1(x_0)$ and $y_u = h_2(x_u)$ be the equations of the top and bottomsides of the profile contour and let (x_0, y_0), (x_u, y_u) be the end points of an arbitrary secant; then the coordinates of the secant center are

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$$x = \frac{x_0 + x_u}{2} \quad \ldots\ldots\ldots \quad (2)$$

$$y = \frac{y_0 + y_u}{2} = \frac{h_1(x_0) + h_2(x_u)}{2} \quad \ldots \quad (3)$$

and the direction of the line element perpendicular to the secant is

$$\frac{dy}{dx} = -\frac{x_0 - x_u}{y_0 - y_u} = -\frac{x_0 - x_u}{h_1(x_0) - h_2(x_u)} \quad . \quad (4)$$

If we consider x_0 and x_u as parameters, then equations (2) to (4) represent the parametric representation of a differential equation which contains the field of all line elements, which are formed by the center normals of all possible profile secants between points of the profile topside and bottomside. By eliminating x_0 and x_u from (2) to (4), we can reduce the differential equation into the usual form

$$\frac{dy}{dx} = F(x,y) \quad \ldots\ldots \quad (5)$$

Equation (4) shows that it is an ordinary differential equation of the first order. Its closed form solution, however, is difficult to obtain because F (x,y) is too complicated or is often not even analytically known. Not much can be said about the degree of the differential equation; it depends for the most part on the profile functions h_1 and h_2 and must therefore be investigated for each case. This will be done later for an example.

The desired camber line is a particular solution of the differential Equation (5). In order to find it, it is necessary to know the initial point. The only one which could be used is the profile trailing edge which is the only known point of the desired camber line. The profile nose then is obtained as the intersection point of the camber line which is specified by the

initial point and the differential equation with the profile contour.

IV. GRAPHICAL OR NUMERICAL SOLUTION

Just like in all cases where there is no closed form solution, the solution of differential Equation (5) is found using a graphical or computational approximation method. Both methods are based on the line element field of the differential equation to be solved. For the purely graphical method, the desired particular curve is drawn through the field. The computation methods use approximate trial solutions, where the direction tangents of certain line elements are involved at the boundaries of selected intervals. In the present case, the direction of the line elements can only be determined graphically. Therefore, the computational method discussed here amounts to a graphical determination of certain ordinate increases at the interval boundaries. Using the computational method of Runge-Kutta which is very well known (see Hütte, Vol. 1, p. 185), it is only necessary to measure the four lengths k_1 to k_4 shown in Figure 3 at the end of the interval. From this the new point of the solution is determined by means of the coordinates

$x_1 = x_0 + h$

$y_1 = y_0 + \Delta y$

(x_0, y_0 = point of the solution curve at the beginning of the interval / 14

(h = interval width)

where $\Delta y = \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

However, before this is done, one must determine the line elements of the differential equation at the points 1 to 4 using graphical methods. For this purpose it is best to use a transparent ruler which has a length division from the center to both sides. A needle or the point of a compass is used to attach it to the points for which one wishes to determine the line elements, so that it can

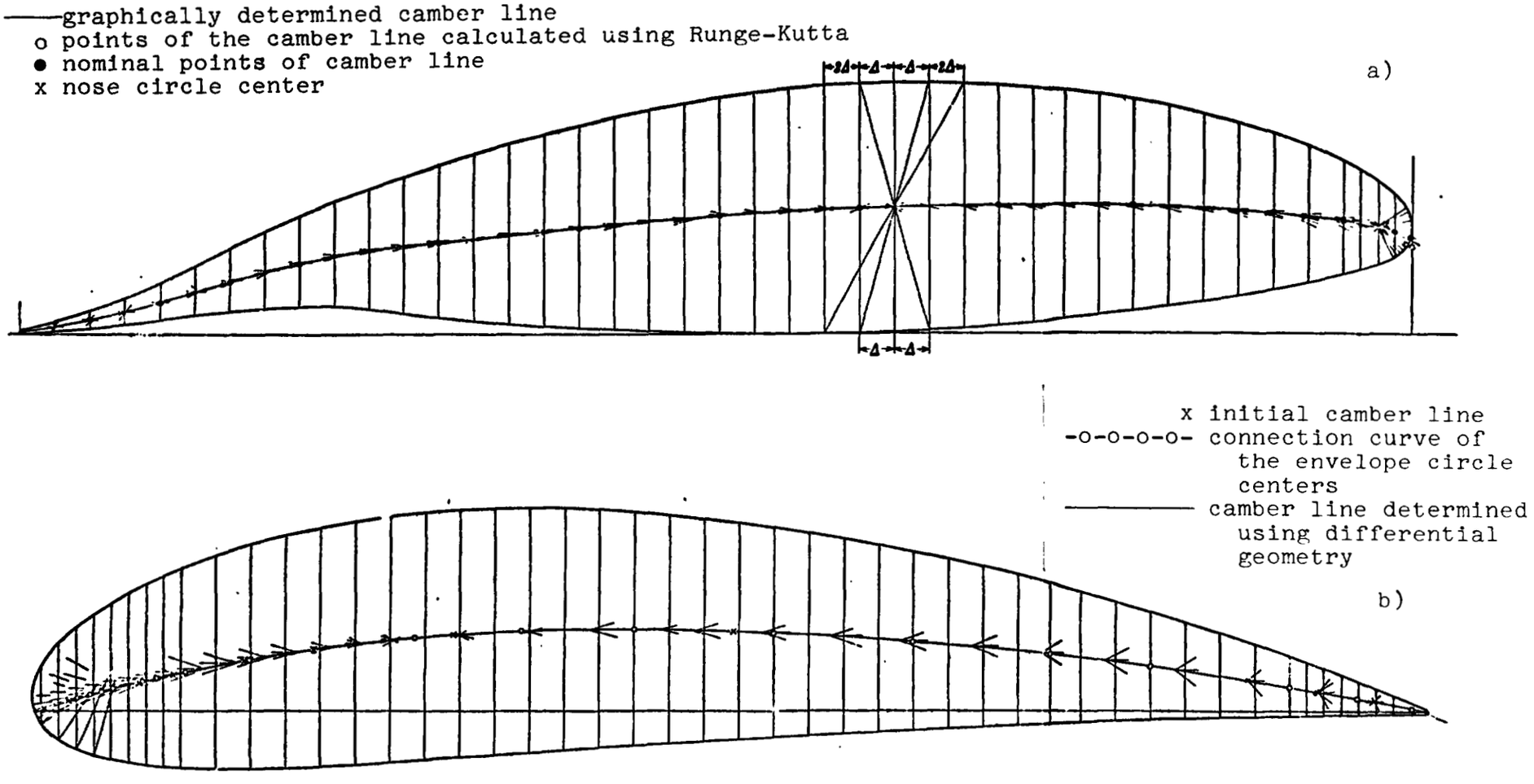
——graphically determined camber line
o points of the camber line calculated using Runge-Kutta
● nominal points of camber line
x nose circle center
a)
x initial camber line
-o-o-o-o- connection curve of the envelope circle centers
——— camber line determined using differential geometry
b)

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Figure 4. Application examples for rigorous graphical camber line determination

a) For a profile of the Type Mustang

b) For a NACA profile

rotate. The ruler is rotated until its lengths along the profile contour are the same. The desired line element is then perpendicular to the ruler direction determined in this way. The accuracy of the curve determination can be increased arbitrarily by increasing the number of computational steps, but at the expense of additional complexity.

It is even simpler to determine the line element field for the purely graphical method, because here one does not have to determine the line elements at certain points, but it is only necessary to introduce a sufficiently dense field. It is appropriate to again select a uniform interval division of the profile in an arbitrarily selected ordinate direction (see Figure 4). If one now wishes to determine profile secants which are bisected along a certain interval line, then it is only necessary to connect one profile point of the topside at a distance of Δx from the division ordinate of the left with the corresponding point on the lower side, or vice versa. At the intersection point with the selected division ordinate, one can draw the line element which is perpendicular to the secant. One can see that the entire line element field covers a relatively narrow strip of the profile center zone. When using the computational method of determining the camber line discussed above, under some conditions it is necessary to make sure that the computational steps have been selected with sufficient density. Otherwise it can happen that one of the points 2 to 4 will lie outside of the line element strip. One other very important property of the field is noticed when constructing the line elements. This is that all elements having a common abscissa represent a ray bundle to a good degree of approximation, whose center is quite close to the selected interval line. Since the camber line is almost always a curve of low curvature, it is permissible to consider it as a connection curve of the centers of the line element bundles. In this way the purely graphical method leads to a rapid and certain determination of camber line (see Example 4, Section VI).

V. THE VICINITY OF THE PROFILE NOSE

The conditions in the immediate vicinity of the profile nose must be discussed separately. In the case of usual profiles in aircraft construction, the nose radius of curvature osculates with the profile contour over quite a wide range. This is the case for the profile without any curvature, and the osculation is symmetric to both sides. Therefore the nose can be determined as the center of this zone with some certainty. In the case of the curve profile, the osculation of the circle of curvature is no longer symmetric. Therefore, the nose cannot be determined directly, which is why the described camber determination method has to start at the trailing edge. Even then the difficulty of constructing the camber in the zone of the nose curvature circle remains, because the line element field inside the circular segment of the contact zone degenerates into a line bundle which passes through the center of the curvature circle. Therefore, it is a matter of feeling in the purely graphical method which line of the bundle is used as the skeleton line inside the circular calotte. The nose point itself has a certain inaccuracy. Therefore, it is recommended to combine the purely graphical method with the computational method for this final point.

The determination of the nose circle center in the case of a curved profile therefore makes up part of determining the line element field. Therefore, the graphical evolute method is best suited for this, i.e., the problem of determining the envelope curve of profile normals in the nose area. One can very accurately establish which bundles are intersected by normals at one point and where this point lies. In the example given in Section VI, one found a nose radius of r_N = 19.7 mm, whereas in reality it was 19.87 mm.

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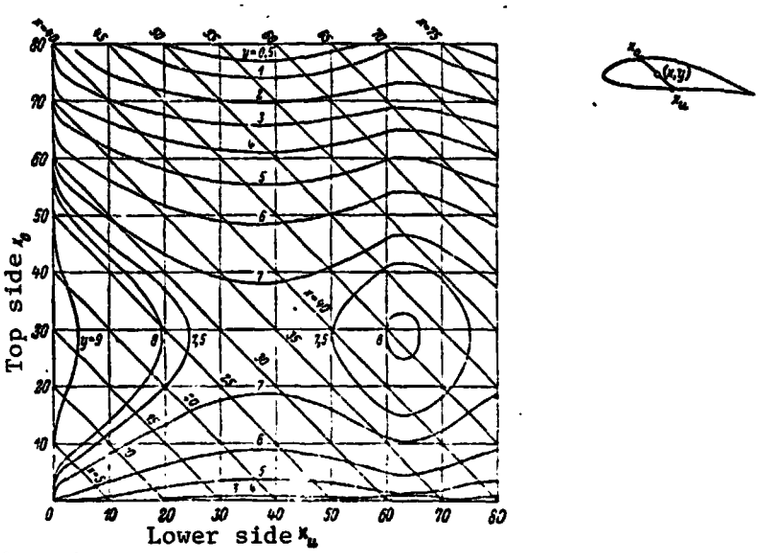

Figure 5. Investigation of the degree of the diferential equation using the number of line elements, which are possible at a point (x, y) in the profile interior; (x_u, x_0) characterize the profile secants.

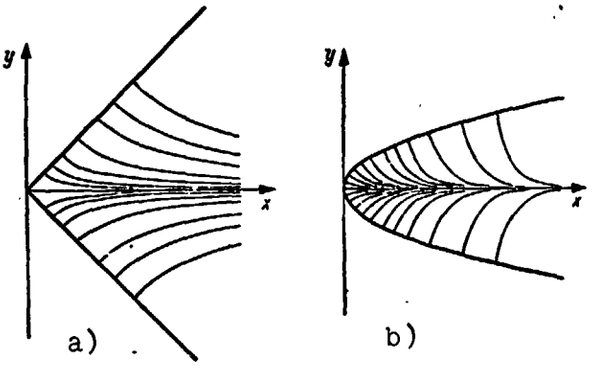

Figure 6. Simple "profiles" with closed form analytical solution of the differential equation of the line element field;

a) Open wedge $y = \pm \ldots$, b) Parabola $y = \pm \sqrt{x}$

In this connection we should point out a frequent error which is committed when constructing curved profiles from symmetric drops and camber lines whereby the nose radius of the drop is assumed to have the same measure as for the deformed profile. If one draws a circle around the center which lies on the initial tangent of a skeleton line with this radius, then this circle will always be too large. However, since the abscissa values of the drop projected on to the oblique camber line and the thicknesses

normal to it are plotted, the curvature circle of the drop deforms into an ellipse. Its semi-axis along the camber line is deformed at a scale of $\frac{1}{\cos\varphi}$, if φ is the initial inclination of the camber line. For the deformed profile, therefore, we have the curvature circle of this ellipse, that is / 15

$$r_{N_1} = r_N \cos\varphi.$$

VI. EXAMPLE

In order to investigate the accuracy of the method discussed, we determine the camber lines of two curved profiles which had been previously constructed from the camber line and the drop. After this we made comparisons. Figure 4 shows that the agreement was very good with the exception of points in the immediate vicinity of the nose. The purely graphic determination method was used in conjunction with the computational method for the last 5% of the profile chord up to the nose.

Figure 5 shows an attempt of determining the degree of the differential equation to be solved for the profile given in Figure 4a. This is simply done by selecting the position coordinates of a line element as parameters, x and y in Equations (2) and (3). In this way one obtains two families of curves $x_0(x_u)$ with the parameters x and y. If these are plotted on top of one another, then each intersection point of two curves results in a value pair x_0, x_u, with a fixed parameter x or y, respectively. In other words this is a secant of the profile which is bisected at x, y. If there is only a single intersection point x_0, x_u, then at the point x, y only one line element exists, and conversely as many line elements exist as intersection points x_0, x_u. In Figure 5, depending on the selection of the parameters x and y, one or two intersection points can be found, but from this fact, one cannot draw any final conclusions regarding the

character of the differential equation and of the particular solution. It is quite possible that the degree of the differential equation will change piecewise, because the thickness function and the camber line function which together determine the line element field can have a piecewise analytical structure which is different.

Apparently the profile shapes which occur in practice are too complicated for an analytical analysis of our problem, which was solved primarily using graphical methods. Therefore, in conclusion we will discuss the analytical camber line determination method using two simple partial profiles. By doing this we would like to respond to questions connected with applying the method to more difficult applications. As Figure 4 shows, there are an arbitrary number of particular solution curves possible in the line element field used for the solution. One has the impression that all of them will flow into the camber line after a short interval. However, without analytical methods, one cannot decide whether the desired camber line represents a singular solution, for example the envelope of all the particular curves, or whether it is only a particular curve. We will investigate this using the example of the trivial "profiles" $y=\pm kx$ and $y=\pm k\sqrt{x}$.* The first profile can be looked upon as the trailing edge of a true profile and the last example can be looked upon as the nose of a true profile. In both cases, the x-axis must result as the "trivial camber line" if we use our solution method. It does not make any difference that these partial profiles are symmetric and also are open towards the back because the method is not subject to any restrictions and the examples show very well what is important.

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* This was suggested by Mrs. Dr. Flügge-Lotz.

1. For the profile $y_0 = kx_0$, $y_u = -kx_u$ Equations (2) to (4) are

$$2x = x_0 + x_u$$
$$2y = k(x_0 - x_u)$$
$$\frac{dy}{dx} = -\frac{x_0 - x_u}{k(x_0 + x_u)}$$

From this we find by elimination of the parameters x_u and x_0

$$\frac{dy}{dx} = -\frac{2y}{k^2 \cdot 2x} = -\frac{1}{k^2} \cdot \frac{y}{x}$$

which is the equation of the desired line element field. It is an ordinary differential equation of the first order and first degree with a singular point at the origin. It can immediately be solved by separating variables and leads to the general solution

$$y = \frac{C}{x^{1/k^2}}.$$

The "camber line" y = 0 of this profile is found as a particular solution by selecting C = 0. Figure 6a shows the general solution for the Case k = 1, i.e. $y = \frac{C}{x}$.

2. For the profile $y_0 = +k\sqrt{x}$, $y_u = -k\sqrt{x}$, we have

$$2x = x_0 + x_u$$
$$2y = k(\sqrt{x_0} - \sqrt{x_u})$$
$$\frac{dy}{dx} = -\frac{x_0 - x_u}{k(\sqrt{x_0} + \sqrt{x_u})} = -\frac{1}{k}(\sqrt{x_0} - \sqrt{x_u})$$

or $\frac{dy}{dx} = -\frac{2y}{k^2}$

as an equation of the line element field. It is again of first order and first degree but does not have any singularities. Its general solution is

$$y = C \cdot e^{-\frac{2}{k^2} \cdot x}$$

and the desired "camber line" $y = 0$ results here as the particular solution for $C = 0$. Figure 6b shows the general solution of the Case $k = 1$, i.e., the parabola $y^2 = x$.

In both examples the camber line is characterized as a particular integral by the fact that it is the asymptote of all other particular solutions. This property of the camber line / 16 can also be transferred to more complicated profile shapes and one has thereby obtained quite a good deal of insight into the mathematical character of the solution.

VII. SUMMARY

A rigorous method for determining the camber line of profiles of unknown curvature is discussed, which is based on the graphical or computational solution of a differential equation. It is not even necessary to explicitly know the equation. The camber line instead is found as the particular solution, which is obtained from the line element field of the differential equation, which can be found by graphical methods. Using an example we show how the method can be rapidly applied with certainty.